

Incoming Precalculus Summer Work

Welcome! The Math Department is so excited that you are taking on the challenge of a course in Precalculus! Precalculus is an in-depth focus on functions and introduction to new families of functions, including trigonometric and exponential. In order to jump into new material as soon as possible, you will need to review topics in the following packet. Some of these concepts may be review to you, and some of them may be new. Do your best!

A few guidelines:

- On this assignment, you may use a calculator. That said, try as much as you can without one.
- If at any time, you need assistance with the topics included here, you may use online resources, such as Khan Academy. Any sources that you use should be cited.
- This assignment will be due on the first full day of school. There will be a non-calculator, graded assessment on these topics within the first two weeks of school.
- Below, you will find a suggested breakdown of the assignment into manageable pieces:
 - Week 1: June 20-June 24
 - Equations of Lines
 - Week 2: June 27-July 1
 - Systems of Equations
 - Week 3: July 5-July 8
 - Exponents
 - Week 4: July 11-July 15
 - Polynomial Operations
 - Week 5: July 18-July 22
 - Radicals
 - Week 6: July 25-July 29
 - SAT Practice
 - Week 6: Aug 1-Aug 5
 - TedTalks and Reflection
- If you have any questions, you can reach me via email at mhain@stpaulsmd.org. Please note that it may take up to 48 hours to receive a response.

I'm looking forward to a great year ahead! Go gators!

-Mrs. Hain

Week 1: Equations of Lines

Slope Intercept form: $y = mx + b$

Vertical Line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal Line: $y = c$ (slope is 0)

Standard Form: $Ax + By = C$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

1. State the slope and y-intercept of the linear equation $5x - 4y = 8$.
2. Find the x-intercept and y-intercept of the equation $2x - y = 5$.
3. Write the equation in standard form: $y = 7x - 5$.
4. Write the equation of the line in slope-intercept form with the following conditions: slope $= -5$, and passes through the point $(-3, -8)$.
5. Write the equation of the line in slope-intercept form with the following conditions: passes through the points $(4, 3)$ and $(7, -2)$.
6. Write the equation of the line in slope-intercept form with the following conditions: x-intercept $= 3$ and y-intercept $= 2$

Week 2: Systems of Equations

$$\begin{aligned}3x + y &= 6 \\ 2x - 2y &= 4\end{aligned}$$

Substitution:

1. Solve 1 equation for 1 variable.
2. Rearrange.
3. Plug into second equation.
4. Solve for the other variable.
5. Plug answer back into an original equation to solve for the second variable.

$$\begin{aligned}y &= 6 - 3x \\ 2x - 2(6 - 3x) &= 4 \\ 2x - 12 + 6 &= 4 \\ 8x &= 16 \\ x &= 2\end{aligned}$$

Elimination:

1. Find opposite coefficients for one variable.
2. Multiply equation(s) by constant(s)
3. Add equations together.
4. Solve for variable.
5. Plug answer back into an original equation to solve for the second variable.

$$\begin{array}{r}6x + 2y = 12 \\ 2x - 2y = 4 \\ \hline 8x = 16 \\ x = 2\end{array}$$

$$\begin{aligned}3(2) + y &= 6 \\ 6 + y &= 6 \\ y &= 0\end{aligned}$$

The answer is (2, 0).

Solve each system of equations. Use any method.

1. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

2. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

3. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Week 3: Exponents

Two Rules of 1: <ul style="list-style-type: none">• $a^1 = a$ <i>Any number raised to the power of one equals itself.</i>• $1^a = 1$ <i>One to any power is one.</i>	Quotient Rule: <ul style="list-style-type: none">• $\frac{a^m}{a^n} = a^{m-n}$ <i>When dividing two powers with the same base, subtract the exponents.</i>
Zero Rule: <ul style="list-style-type: none">• $a^0 = 1$ <i>Any nonzero number raised to the power of zero is one.</i>	Power Rule: <ul style="list-style-type: none">• $(a^m)^n = a^{mn}$ <i>When a power is raised to another power, multiply the exponents.</i>
Product Rule: <ul style="list-style-type: none">• $a^m \cdot a^n = a^{m+n}$ <i>When multiplying two powers that have the same base, add the exponents.</i>	Negative Exponents: <ul style="list-style-type: none">• $a^{-n} = \frac{1}{a^n}$• $\frac{1}{a^{-n}} = a^n$ <i>Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.</i>

Express the following in simplest form. Answers should not have any negative exponents.

1. $5a^0$

3. $\frac{2ef^{-1}}{e^{-1}}$

2. $\frac{3c}{c^{-1}}$

4. $\frac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify the following.

5. $3m^2 \cdot 2m$

6. $(a^3)^2$

7. $(-b^3c^4)^5$

8. $4m(3a^2m)$

Week 4: Polynomial Operations

To add/subtract polynomials, combine like terms:

$$\begin{aligned} &8x - 3y + 6 - (6y + 4x - 9) \\ &= 8x - 3y + 6 - 6y - 4x + 9 \\ &= 4x - 9y + 15 \end{aligned}$$

To multiply polynomials, use FOIL or the distributive property.

$$\begin{aligned} &(3x - 2)(x + 4) \\ &= 3x^2 + 12x - 2x - 8 \\ &= 3x^2 + 10x - 8 \end{aligned}$$

When factoring polynomials, use the following techniques:

- Find the GCF first
- Check if it is a difference of two cubes
- Use the diamond method
- If there are four terms, factor by grouping

1. Simplify the following:

a. $3x^3 + 9 + 7x^2 - x^3$

b. $7m - 6 - (2m + 5)$

2. Multiply the following:

a. $(3a + 1)(a - 2)$

b. $(c - 5)^2$

c. $(5x + 7y)(5x - 7y)$

3. Factor completely:

a. $z^2 + 4z - 12$

b. $6 - 5x - x^2$

c. $2k^2 + 2k - 60$

Week 5: Radicals

To **simplify** means that no radicand (inside number) has a perfect square factor and there is no radical in the denominator (rationalize).

Recall, the **product property**: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

And the **quotient property**: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples:

Simplify $\sqrt{24} = \sqrt{6} \cdot \sqrt{4} = 2\sqrt{6}$

Rationalize $\frac{1}{\sqrt{2}}$

$$\begin{aligned}\frac{1}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{2}}\right) \\&= \frac{\sqrt{2}}{\sqrt{4}} \\&= \frac{\sqrt{2}}{2}\end{aligned}$$

Simplify or rationalize each of the following:

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt{49m^2n^8}$

4. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

5. $\frac{5}{\sqrt{3}}$

Week 6: SAT Practice

Answer the following questions using the skills you completed in previous weeks of this packet and previous classes that you have taken.

The recommended daily calcium intake for a 20-year-old person is 1,000 milligrams (mg). One cup of milk contains 299 mg of calcium and one cup of juice contains 261 mg of calcium. Which of the following inequalities represents the possible number of cups of milk, m , and cups of juice, j , a 20-year-old person could drink in a day to meet or exceed the recommended daily calcium intake from these drinks alone?

- A) $299m + 261j \geq 1,000$
- B) $299m + 261j > 1,000$
- C) $\frac{299}{m} + \frac{261}{j} \geq 1,000$
- D) $\frac{299}{m} + \frac{261}{j} > 1,000$

A company's manager estimated that the cost C , in dollars, of producing n items is $C = 7n + 350$. The company sells each item for \$12. The company makes a profit when the total income from selling a quantity of items is greater than the total cost of producing that quantity of items. Which of the following inequalities gives all possible values of n for which the manager estimates that the company will make a profit?

- A) $n < 70$
- B) $n < 84$
- C) $n > 70$
- D) $n > 84$

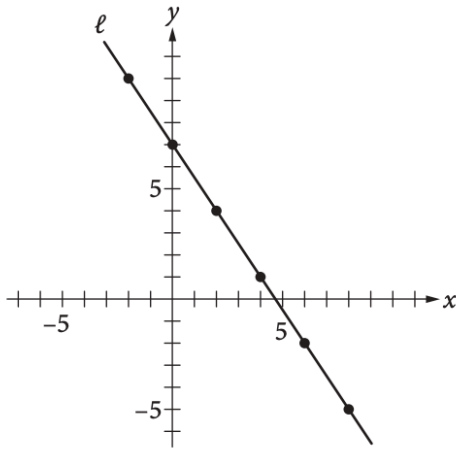
The average number of students per classroom, y , at Central High School can be estimated using the equation $y = 0.8636x + 27.227$, where x represents the number of years since 2004 and $x \leq 10$. Which of the following statements is the best interpretation of the number 0.8636 in the context of this problem?

- A) The estimated average number of students per classroom in 2004
- B) The estimated average number of students per classroom in 2014
- C) The estimated yearly decrease in the average number of students per classroom
- D) The estimated yearly increase in the average number of students per classroom

If $\frac{2}{a-1} = \frac{4}{y}$, and $y \neq 0$ where $a \neq 1$, what is y in terms of a ?

- A) $y = 2a - 2$
- B) $y = 2a - 4$
- C) $y = 2a - \frac{1}{2}$
- D) $y = \frac{1}{2}a + 1$

Line ℓ is graphed in the xy -plane below.



If line ℓ is translated up 5 units and right 7 units, then what is the slope of the new line?

- A) $\frac{2}{5}$
- B) $-\frac{3}{2}$
- C) $-\frac{8}{9}$
- D) $-\frac{11}{14}$

If $a^{-\frac{1}{2}} = x$, where $a > 0$ and $x > 0$, which of the following equations gives a in terms of x ?

- A) $a = \frac{1}{\sqrt{x}}$
- B) $a = \frac{1}{x^2}$
- C) $a = \sqrt{x}$
- D) $a = -x^2$

If $y = x^3 + 2x + 5$ and $z = x^2 + 7x + 1$, what is $2y + z$ in terms of x ?

- A) $3x^3 + 11x + 11$
- B) $2x^3 + x^2 + 9x + 6$
- C) $2x^3 + x^2 + 11x + 11$
- D) $2x^3 + 2x^2 + 18x + 12$

At a primate reserve, the mean age of all the male primates is 15 years, and the mean age of all female primates is 19 years. Which of the following must be true about the mean age m of the combined group of male and female primates at the primate reserve?

- A) $m = 17$
- B) $m > 17$
- C) $m < 17$
- D) $15 < m < 19$

Week 7: TedTalks and Reflections

Watch the following Ted Talks on math and problem solving and answer the reflection questions after.

Roger Antonsen: "Math is the hidden secret to understanding the world"

https://www.ted.com/talks/roger_antonsen_math_is_the_hidden_secret_to_understanding_the_world_-_t-1011239

Questions:

1. Talk about a time that you considered different perspectives. (This can be any time—not just math!)
2. How can you apply this perspective to math class?

Jeff Dekofsky: Is math discovered or invented?

https://www.ted.com/talks/jeff_dekofsky_is_math_discovered_or_invented#t-1390

1. What do you think—was math discovered or invented?
2. What do you notice about most of the mathematicians mentioned in the video?